MATH4010 Functional Analysis (2020-21): Homework 5. Deadline: 19 Nov 2020

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.
✠ Each answer paper must include your name and student ID.

1. Let (x_n) and (y_n) be the sequences in a Hilbert space H. Suppose that the limits $\lim\|x_n\|, \lim\|y_n\|$ and $\lim\|\frac{x_n+y_n}{2}\|$ exist and are equal. Show that if (x_n) is convergent, then so is (y_n).

2. Fix an element $z \in H$. Define a linear functional φ on H by $\varphi(x) = (x, z)$.
   (i) Show that $\|\varphi\| = \|z\|$.
   (ii) Let $w \in H$. Find $dist(w, \ker\varphi)$, the distance between the element w and $\ker\varphi$. (the answer is in terms of w and z.)
   (iii) Let $H = L^2(\mathbb{T})$ and φ be the functional on H given by $\varphi(f) := \int_{\mathbb{T}} f(z)dz$ for $f \in H$. Let $g \in H$. Find the element $h \in \ker\varphi$ such that $\|g - h\| = dist(g, \ker\varphi)$.

$* * *$ **End** $* * *$